

June 1994 (revised March 1995)

PAR-LPTHE 94/24

hep-ph/9406349

COMMENTS ON THE STANDARD MODEL OF ELECTROWEAK INTERACTIONS.B. Machet,^{1 2}

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Abstract: The Standard Model of electroweak interactions is shown to include a gauge theory for the observed scalar and pseudoscalar mesons. This is done by exploiting the consequences of embedding the $SU(2)_L \times U(1)$ group into the chiral group of strong interactions and by considering explicitly the Higgs boson and its three companions inside the standard scalar 4-plet as composite. No extra scale of interaction is needed. Quantizing by the Feynman path integral reveals how, in the ‘Nambu Jona-Lasinio approximation’, the quarks and the Higgs boson become unobservable, and the theory anomaly-free. Nevertheless, the ‘anomalous’ couplings of the pseudoscalar mesons to gauge fields spring again from the constraints associated with their compositeness itself. This work is the complement of ref. [1], where the leptonic sector was shown to be compatible with a purely vectorial theory and, consequently, to be also anomaly-free. The bond between quarks and leptons loosens.

PACS: 11.15.Ex, 11.30.-j, 11.30.Rd, 11.40.Ha, 12.15.-y, 12.15.Ff, 12.38.Cy, 12.38.Lg, 12.60.Nz, 12.60.Rc, 13.20.-v, 13.40.Hq, 14.40.-n, 14.80.Bn, 14.80.Cp.



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1 Introduction.

Leaving aside gravitation, the interactions of fermions split into two categories: strong and electroweak; no true unification is yet achieved between the two. In the work [1], we showed that the effective $V - A$ structure of leptonic weak interactions can be deduced from a purely vectorial theory, and that the ‘right-handed’ neutrino becomes unobservable as asymptotic state. I focus now on the quark sector, where both types of interactions are at work. I present an attempt to partially fill the gap between them, with the (restricted) meaning that some characteristics, attributed before either to one or to the other, are now visualized from a unified point of view.

Quantum Chromodynamics, the would-be candidate for a theory of strong interactions of quarks and gluons (see for example [2]), will not be dealt with here. It fails, up to now, to describe the basics of low energy interactions between hadrons, which cannot themselves be accounted for as asymptotic states. As the point cannot be settled whether this is a failure of the theory or a result of our incapacity to compute, I will consider QCD to be irrelevant at low energy.

Going in some way backwards in time, I will instead be concerned here with the link between one fundamental aspect underlying strong interactions, chiral symmetry (see for example [3]), including the diagonal ‘flavour group’ at the root of the constituent quark model, and the gauge group of electroweak interactions [4]. I give a new description of the breaking of chiral symmetry, leading in particular to a new interpretation of the associated Goldstone bosons [5]; the gauge group being identified with a subgroup of the chiral group of strong interactions, the latter become now electroweak eigenstates, mixing, into the same multiplets, scalar and pseudoscalar mesons: electroweak and ‘strong’ eigenstates are linked together by relations which generalize those existing, for example, between the K_1, K_2 mesons and K^0, \bar{K}^0 . This identification has numerous consequences, only a part of which is sketched here; I will focus on the resulting classification of the scalar and pseudoscalar mesons into $SU(2)_L \times U(1)$ multiplets, which yield new mass relations and degeneracies. In particular the question of the mass of the η' meson can find a natural solution outside the intricacies of the QCD vacuum [6].

By naming the η' without adding any ‘techni-’like prefix, I already mean that the above mentioned scalar and pseudoscalar fields are directly related to—in fact just linear combinations of—the daily observed scalar and pseudoscalar mesons, and not to a higher scale of interactions like in technicolour theories [7]. The accent is specially put on two processes: the leptonic decays of the pseudoscalar mesons and their ‘anomalous’ couplings to two gauge fields.

Unlike in the traditional scheme of chiral symmetry breaking, we do not expect N^2 (N = number of flavours of quarks) ‘light’ particles. In general, mesons can be given masses in an $SU(2)_L \times U(1)$ invariant way, without any reference to the parameters called ‘quark masses’, and only three among the above electroweak eigenstates become the longitudinal components of the three massive gauge fields. The different mass scales appear directly at the mesonic level in the Lagrangian.

Another original aspect of this model is the relation between fermions and mesons that results from its quantization. Nature shows, in an, up to now, unambiguous way, that they do not co-exist as asymptotic states; this is akin to saying that quarks, as asymptotic states, have infinite masses. This is what occurs here, by the Feynman path integral quantization, when, in the so-called ‘Nambu Jona-Lasinio’ approximation [8], corresponding to keeping the leading order in a development in inverse powers of the number of flavours, the three longitudinal components of the massive W ’s and the Higgs boson (forming the usual scalar sector) are explicitly considered to be composite. Two ingredients concur for that:

- first, the necessity, when integrating on both fermions and bound states which are non-independent

degrees of freedom, to explicitly introduce constraints in the path integral; they can be exponentiated into an effective Lagrangian;

- secondly, the freezing of the fermionic degrees of freedom themselves when the Higgs boson gets a non-vanishing vacuum expectation value: an infinite mass for the quarks appears in the above effective Lagrangian. In addition, the Higgs boson becomes itself infinitely massive and unobservable.

The result is a gauge, unitary theory of mesons and gauge fields, the latter being as usual the massless photon and the three massive W 's. I show that it is anomaly-free but that, nevertheless, 'anomalous' couplings of the pseudoscalar mesons to gauge fields are rebuilt from the constraints, with subtleties which will be evoked.

The mass of the Higgs boson being infinite, the massive gauge bosons are expected to be strongly interacting [9], which is just another facet of strong interactions that usual pseudoscalar and vector mesons undergo. A bridge thus starts being built between the weak and strong sector.

Due to the complexity of the effective Lagrangian originating from the constraints, I only study renormalizability in the already mentioned 'Nambu Jona Lasinio approximation' [8], in which precisely the sole bound states propagate.

Similarly, for the sake of simplicity, I only study the 4-flavour case, leaving temporarily aside phenomena like the violation of CP , and the largest part of the argumentation is performed in an abelian model which has all the characteristic and interesting features without the unnecessary intricacies linked with a non-abelian group.

The title of this work is justified by the conservative point of view adopted here. The only *addenda* with respect to the Glashow-Salam-Weinberg model are mainly conceptual, since they concern the embedding of the gauge group into the chiral group, and considering the standard scalar multiplet, *i.e.* the Higgs boson and its three companions, as composite. The consequences are, despite what one could think, large, and not so standard, and have effects on many basic aspects of the electroweak interactions of mesons. By establishing a link between two, up to now disconnected, groups of symmetry, one hopes that they will provide a new insight into a possible unification with the strong interactions.

2 Embedding the gauge group into the chiral group.

2.1 The 'standard' picture of chiral symmetry breaking.

Let N be the number of quark flavours. The group of chiral symmetry is $\mathcal{G}_\chi = U(N)_L \times U(N)_R$, where the subscripts 'L' and 'R' mean 'left' and 'right' respectively. In the standard picture [3, 10], it is broken down to the diagonal $U(N)$ of flavour, which, in the $N = 3$ case, includes the $SU(3)$ of Gell-Mann [11]. The N^2 corresponding Goldstone bosons are identified with the N^2 pseudoscalar mesons. In the quark model, chiral symmetry breaking is triggered by 'quark condensation'

$$\langle \bar{\Psi}\Psi \rangle = N\mu^3. \quad (1)$$

The non-zero and very different masses of the 'Goldstones' are accounted for by putting by hand different quark masses in the Lagrangian, which produce an explicit breaking of chiral symmetry. Low energy theorems combined with the 'Partially Conserved Axial Current' (PCAC) hypothesis, link quark masses m , the scale of breaking μ , the mass of the Goldstones M_G and their weak decay constants f_G by relations of the type [12]

$$m\mu^3 = -\kappa f_G^2 M_G^2, \quad (2)$$

where κ is a numerical group factor of the order of unity. The fact that the η' meson has a much heavier mass than the pion has been a long standing problem [6].

2.2 The standard picture of electroweak symmetry breaking.

The Standard Model of electroweak interactions by Glashow, Salam and Weinberg, [4][13] involves four (real) scalar fields, including the Higgs boson. The gauge symmetry is broken by the latter acquiring a non-vanishing vacuum expectation value in the vacuum

$$\langle H \rangle = v/\sqrt{2}. \quad (3)$$

The three companions of the Higgs are the Goldstone bosons of the broken $SU(2)$ symmetry; they can be gauged into the third polarizations of the three massive gauge fields, the W gauge bosons. The zero mass of the photon is preserved, corresponding to an unbroken $U(1)$ group for pure electromagnetism.

The coupling of the gauge fields to the quarks of electric charge $-1/3$ occurs through a mixing matrix, the Cabibbo matrix [14] in the 4-quark case (generalized to the Kobayashi-Maskawa matrix [15] in the 6-quark case). The introduction of the Cabibbo mixing angle being one of the most well confirmed fact, I will not question its existence nor that of a unitary mixing matrix.

2.3 The embedding.

The gauge group $\mathcal{G}_s = SU(2)_L \times U(1)$ acts on the fermions (quarks) and on the gauge fields. I recall that, for the sake of simplicity, we only deal here with the 4-quark case. The reader can understand how the generalization proceeds.

2.3.1 Identifying the generators of the gauge group as 4×4 matrices of $U(4)$.

We work in the fermion basis

$$\Psi = \begin{pmatrix} u \\ c \\ d \\ s \end{pmatrix}. \quad (4)$$

The identification of the generators of \mathcal{G}_s as 4×4 matrices proceeds as follows:

$$\mathbb{T}_L^+ = \mathbb{T}_L^1 + i\mathbb{T}_L^2 = \frac{1-\gamma_5}{2} \mathbb{C}, \quad \mathbb{T}_L^- = \mathbb{T}_L^1 - i\mathbb{T}_L^2 = \frac{1-\gamma_5}{2} \mathbb{C}^\dagger, \quad \mathbb{T}_L^3 = \frac{1}{2} \frac{1-\gamma_5}{2} \mathbb{N}; \quad (5)$$

- for $U(1)_L$:

$$\mathbb{Y}_L = \frac{1-\gamma_5}{2} \mathbb{Y} = \frac{1}{6} \frac{1-\gamma_5}{2} \mathbb{I}; \quad (6)$$

- for $U(1)_R$:

$$\mathbb{Y}_R = \frac{1+\gamma_5}{2} \mathbb{Y} = \frac{1+\gamma_5}{2} \mathbb{Q}, \quad (7)$$

where

$$\mathbb{C} = \left(\begin{array}{c|c} 0 & \mathbb{C} \\ \hline 0 & 0 \end{array} \right); \quad \mathbb{N} = \left(\begin{array}{c|c} 1 & 0 \\ \hline 0 & -1 \end{array} \right); \quad \mathbb{Q} = \left(\begin{array}{c|c} \frac{2}{3} & 0 \\ \hline 0 & -\frac{1}{3} \end{array} \right). \quad (8)$$

\mathbf{C} is the customary 2×2 Cabibbo mixing matrix [14]. \mathbb{I} is the unit matrix. We have the usual Gell-Mann-Nishijima relation

$$\mathbb{Y} = \mathbb{Y}_R + \mathbb{Y}_L = \mathbb{Q} - \mathbb{T}_L^3 = \mathbb{Q}_R + \mathbb{Y}_L. \quad (9)$$

It is an essential feature that not only the \mathbb{T} 's and the right and left \mathbb{Y} 's have the usual $SU(2)_L \times U(1)$ commutation relations, but also that the \mathbb{T}_L 's and \mathbb{Y}_L form a *matrix algebra* by themselves: any product of two among the four stays in the set (this property makes the 'left' part of the gauge group isomorphic to $U(2)$). We have indeed:

$$\begin{aligned} \{3\mathbb{Y}_L, \mathbb{T}_L^3\} &= \mathbb{T}_L^3, & \{3\mathbb{Y}_L, \mathbb{T}_L^+\} &= \mathbb{T}_L^+, & \{3\mathbb{Y}_L, \mathbb{T}_L^-\} &= \mathbb{T}_L^-, \\ \{\mathbb{T}_L^3, \mathbb{T}_L^3\} &= 3\mathbb{Y}_L, & \{\mathbb{T}_L^3, \mathbb{T}_L^+\} &= 0, & \{\mathbb{T}_L^3, \mathbb{T}_L^-\} &= 0, \\ \{\mathbb{T}_L^+, \mathbb{T}_L^-\} &= 6\mathbb{Y}_L. \end{aligned} \quad (10)$$

This property allows scalar representations of composite quark-antiquark fields, mixing scalars and pseudoscalars; indeed, because of the γ_5 matrix appearing in the 'left' gauge generators, their action on composite fermion operators of the form $\bar{\Psi}\mathcal{O}\Psi$ will involve both their commutators and their anticommutators with \mathcal{O} . It will be thoroughly exploited in the rest of the paper; for the moment, we can immediately see that the composite scalar multiplet

$$\Phi = (H, \phi^3, \phi^+, \phi^-) = \frac{v}{N\mu^3} \bar{\Psi} \left(\frac{1}{\sqrt{2}} \mathbb{I}, \frac{1}{\sqrt{2}} \gamma_5 \mathbb{N}, \gamma_5 \mathbf{C}, \gamma_5 \mathbf{C}^\dagger \right) \Psi, \quad (11)$$

where the conventions are

$$\phi^+ = \frac{\phi_1 + i\phi_2}{\sqrt{2}}, \quad \phi^- = \frac{\phi_1 - i\phi_2}{\sqrt{2}}, \quad (12)$$

is isomorphic to the standard scalar multiplet of the Glashow-Salam-Weinberg model. Here, H is real and the ϕ 's purely imaginary; we shall also use the real fields $\vec{\phi}$ such that $\vec{\phi} = i \vec{\phi}$. The action of \mathcal{G}_s on Φ , deduced from that on the fermions, reads:

$$\mathbb{T}_L^i \cdot \phi_j = -\frac{1}{2} (i \varepsilon_{ijk} \phi_k + \delta_{ij} H) \quad (13)$$

$$\mathbb{T}_L^i \cdot H = -\frac{1}{2} \phi_i, \quad (14)$$

showing that it is as usual the sum of two representations $1/2$ of $SU(2)_L$. The action of $U(1)$ is deduced from the Gell-Mann-Nishijima relation and that, trivial of the charge operator \mathbb{Q} . This means in particular that the mechanism of mass generation for the gauge bosons stays unaltered.

By this embedding, chiral and gauge symmetry breaking appear naturally as two aspects of the same phenomenon, since

$$\langle H \rangle = v/\sqrt{2} \quad (15)$$

is now equivalent to

$$\langle \bar{\Psi}\Psi \rangle = N\mu^3. \quad (16)$$

It is *not* our goal here to trace the origin of this phenomenon, that we take for granted and consider to be a constraint on the system. We shall only study its consequences and show its consistency with the existence of bound states. This makes our work much less ambitious, in particular, than that of Nambu and Jona-Lasinio [8], the analogies and differences with which we shall stress in the following (see section 4.2).

Strictly speaking, we could at that point proceed with the study of the Standard Model, taking only into account the modification due to the compositeness of the scalar multiplet. We shall indeed show later that the pseudoscalar partners of the Higgs boson behave like linear combinations of some of the *observed* pseudoscalar mesons, and *not* like ‘technimesons’ which would correspond to another higher scale of interactions [7] (and I will show that the latter is not needed here). However, $2N^2 - 3$ scalars and pseudoscalars composed from a quark-antiquark pair are still missing; this is why I will first continue to exploit the very peculiar group structure of the model and show how the other mesons fit into this framework. I stay at the level of simple group theory arguments, postponing dynamical considerations to a later part of the paper.

2.4 More structure.

I show that the existence of another scalar 4-plet of the group \mathcal{G}_s is related with the presence of a $(SU(2)_L \times U(1)) \times (SU(2)_L \times U(1))$ group of symmetry. The mixing angle appears as controlling the embedding of \mathcal{G}_s into the latter.

Let \mathcal{P}_1 and \mathcal{P}_2 be the 2×2 orthogonal projector matrices

$$\mathcal{P}_1 = \frac{1}{2} \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}, \quad \mathcal{P}_2 = \frac{1}{2} \begin{pmatrix} 1 & -i \\ i & 1 \end{pmatrix}; \quad (17)$$

they satisfy

$$\mathcal{P}_1^2 = \mathcal{P}_1, \quad \mathcal{P}_2^2 = \mathcal{P}_2, \quad \mathcal{P}_1 \mathcal{P}_2 = \mathcal{P}_2 \mathcal{P}_1 = 0. \quad (18)$$

The 6 matrices built from the \mathcal{P} ’s

$$t^+ = \left(\begin{array}{c|c} 0 & \mathcal{P} \\ \hline 0 & 0 \end{array} \right), \quad t^- = \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathcal{P} & 0 \end{array} \right), \quad t^3 = \frac{1}{2} \left(\begin{array}{c|c} \mathcal{P} & 0 \\ \hline 0 & -\mathcal{P} \end{array} \right) \quad (19)$$

form the generators of two $SU(2)$ commuting groups, the first associated with \mathcal{P}_1 , the second associated with \mathcal{P}_2 . We introduce in addition the 4 matrices n_1, n_2, q_1, q_2 according to

$$n = \left(\begin{array}{c|c} \mathcal{P} & 0 \\ \hline 0 & \mathcal{P} \end{array} \right), \quad q = \left(\begin{array}{c|c} \frac{2}{3}\mathcal{P} & 0 \\ \hline 0 & -\frac{1}{3}\mathcal{P} \end{array} \right), \quad (20)$$

to which we associate the ‘hypercharge’ generators

$$y_L = \frac{1}{6}n_L, \quad y_R = q_R. \quad (21)$$

(\vec{t}_{L1}, y_1) form a first $SU(2)_L \times U(1)$ group, \mathcal{G}_1 , (\vec{t}_{L2}, y_2) form a second $SU(2)_L \times U(1)$ group, \mathcal{G}_2 . The commutation relations of any $SU(2)$ group are unaltered if we perform a rotation between the t^1 and t^2 generators, or, equivalently, if we replace t^+ by $e^{i\alpha} t^+$ and t^- by $e^{-i\alpha} t^-$, leaving t^3 unchanged; we call the corresponding group $\mathcal{G}(\alpha)$. It is now easy to see that the group of the Standard Model, \mathcal{G}_s can be written symbolically

$$\mathcal{G}_s = \mathcal{G}_1(-\theta_c) + \mathcal{G}_2(\theta_c), \quad (22)$$

where θ_c is the Cabibbo angle. By the above formula, I mean the following relations between the generators of the groups:

$$\begin{aligned} \mathbb{T}^3 &= t_1^3 + t_2^3, \quad \mathbb{T}^+ = e^{-i\theta_c} t_1^+ + e^{i\theta_c} t_2^+, \quad \mathbb{T}^- = e^{i\theta_c} t_1^- + e^{-i\theta_c} t_2^-, \\ \mathbb{Y}_L &= y_{1L} + y_{2L}, \quad \mathbb{Y}_R = y_{1R} + y_{2R}. \end{aligned} \quad (23)$$

θ_c now controls the embedding of the gauge group into the product of two $SU(2)_L \times U(1)$ ’s.

2.4.1 Another scalar multiplet.

Let us call Φ_1 and Φ_2 the two scalar multiplets

$$\Phi_1 = \frac{\sigma_1}{N} \bar{\Psi}(n_1, \gamma_5 \vec{t}_1) \Psi, \quad \Phi_2 = \frac{\sigma_2}{N} \bar{\Psi}(n_2, \gamma_5 \vec{t}_2) \Psi. \quad (24)$$

σ_1 and σ_2 have dimension $mass^{-2}$. \mathcal{G}_1 has a vanishing action on Φ_2 , and \mathcal{G}_2 on Φ_1 .

It is easy to check that we obtain other representations by performing the same rotations on the generators appearing in the Φ 's as in the above subsection; more precisely, for example,

$$\Phi_1(\alpha) = \frac{\sigma}{N} \bar{\Psi} \left(\frac{1}{\sqrt{2}} n_1, \frac{1}{\sqrt{2}} \gamma_5 t_1^3, e^{i\alpha} \gamma_5 t_1^+, e^{-i\alpha} \gamma_5 t_1^- \right) \Psi \quad (25)$$

is a representation of $\mathcal{G}_1(\alpha)$, invariant by \mathcal{G}_2 . We can now identify the basic representation Φ as

$$\Phi = \Phi_1(-\theta_c) + \Phi_2(\theta_c), \quad (26)$$

and find another scalar multiplet representation of \mathcal{G}_s

$$\begin{aligned} \Xi &= \Phi_1(-\theta_c) - \Phi_2(\theta_c) = \\ &= \frac{\xi}{N} \bar{\Psi} \left(\frac{1}{\sqrt{2}} (n_1 - n_2), \frac{1}{\sqrt{2}} \gamma_5 (t_1^3 - t_2^3), \gamma_5 (e^{-i\theta_c} t_1^+ - e^{i\theta_c} t_2^+), \gamma_5 (e^{i\theta_c} t_1^- - e^{-i\theta_c} t_2^-) \right) \Psi. \end{aligned} \quad (27)$$

ξ has dimension $mass^{-2}$. Note that this occurs despite the fact that $\mathcal{G}_1(-\theta_c) - \mathcal{G}_2(\theta_c)$ is *not* a group. The structure at the origin of Ξ is actually a $Z(2)$ gradation of the gauge group.

To have a more explicit view in terms of quarks, note that

$$e^{-i\theta_c} \mathcal{P}_1 - e^{i\theta_c} \mathcal{P}_2 = -i \begin{pmatrix} \sin \theta_c & -\cos \theta_c \\ \cos \theta_c & \sin \theta_c \end{pmatrix}; \quad \mathcal{P}_1 - \mathcal{P}_2 = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}. \quad (28)$$

2.5 The last multiplets.

With four flavours of quarks, we expect sixteen scalar and sixteen pseudoscalar fermion-antifermion pairs, of which we have up to now exhibited only eight. I now show how the last twenty-four appear.

2.5.1 Two more scalar 4-plets.

Let \mathcal{P}^+ and \mathcal{P}^- be the two 2×2 matrices

$$\mathcal{P}_+ = \frac{1}{2} \begin{pmatrix} i & 1 \\ 1 & -i \end{pmatrix}, \quad \mathcal{P}_- = \frac{1}{2} \begin{pmatrix} -i & 1 \\ 1 & i \end{pmatrix}. \quad (29)$$

They are nilpotent:

$$\mathcal{P}_+^2 = \mathcal{P}_-^2 = 0. \quad (30)$$

We have the relations

$$\mathcal{P}_+ \mathcal{P}_- = \mathcal{P}_1, \quad \mathcal{P}_- \mathcal{P}_+ = \mathcal{P}_2,$$

$$\begin{aligned}
\mathcal{P}_1 \mathcal{P}_+ &= \mathcal{P}_+ \mathcal{P}_2 = \mathcal{P}_+, \\
\mathcal{P}_2 \mathcal{P}_- &= \mathcal{P}_- \mathcal{P}_1 = \mathcal{P}_-, \\
\mathcal{P}_1 \mathcal{P}_- &= \mathcal{P}_- \mathcal{P}_2 = \mathcal{P}_2 \mathcal{P}_+ = \mathcal{P}_+ \mathcal{P}_1 = 0.
\end{aligned} \tag{31}$$

The four 2×2 matrices $\mathcal{P}_1, \mathcal{P}_2, \mathcal{P}_+, \mathcal{P}_-$ span a $U(2)$ algebra.

From the last two, we can build two more, complex-conjugate, 4-dimensional multiplets Δ and Θ , representations of \mathcal{G}_s , according to

$$\begin{aligned}
\Delta &= \frac{\rho}{N} \times \\
\bar{\Psi} \left(\frac{1}{\sqrt{2}} \left(\begin{array}{c|c} e^{-i\theta_c} \mathcal{P}_+ & 0 \\ \hline 0 & e^{+i\theta_c} \mathcal{P}_+ \end{array} \right), \frac{1}{\sqrt{2}} \gamma_5 \left(\begin{array}{c|c} e^{-i\theta_c} \mathcal{P}_+ & 0 \\ \hline 0 & -e^{+i\theta_c} \mathcal{P}_+ \end{array} \right), \gamma_5 \left(\begin{array}{c|c} 0 & \mathcal{P}_+ \\ \hline 0 & 0 \end{array} \right), \gamma_5 \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathcal{P}_+ & 0 \end{array} \right) \right) \Psi,
\end{aligned} \tag{32}$$

$$\begin{aligned}
\Theta &= \frac{\omega}{N} \times \\
\bar{\Psi} \left(\frac{1}{\sqrt{2}} \left(\begin{array}{c|c} e^{+i\theta_c} \mathcal{P}_- & 0 \\ \hline 0 & e^{-i\theta_c} \mathcal{P}_- \end{array} \right), \frac{1}{\sqrt{2}} \gamma_5 \left(\begin{array}{c|c} e^{+i\theta_c} \mathcal{P}_- & 0 \\ \hline 0 & -e^{-i\theta_c} \mathcal{P}_- \end{array} \right), \gamma_5 \left(\begin{array}{c|c} 0 & \mathcal{P}_- \\ \hline 0 & 0 \end{array} \right), \gamma_5 \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathcal{P}_- & 0 \end{array} \right) \right) \Psi.
\end{aligned} \tag{33}$$

ρ and ω have dimension $mass^{-2}$.

2.5.2 A trivial doubling.

Is is trivial to uncover the last four 4-plets of scalars by performing on $\Phi, \Xi, \Delta, \Theta$ a γ_5 transformation. We get from Φ (eq. (11))

$$\tilde{\Phi} = \frac{\varsigma}{N} \bar{\Psi} \left(\frac{1}{\sqrt{2}} \gamma_5 \mathbb{I}, \frac{1}{\sqrt{2}} \mathbb{N}, \mathbb{C}, \mathbb{C}^\dagger \right) \Psi, \tag{34}$$

from Ξ (eq. (27))

$$\tilde{\Xi} = \frac{\tilde{\xi}}{N} \bar{\Psi} \left(\frac{1}{\sqrt{2}} (n_1 - n_2) \gamma_5, \frac{1}{\sqrt{2}} (t_1^3 - t_2^3), (e^{-i\theta_c} t_1^+ - e^{i\theta_c} t_2^+), (e^{i\theta_c} t_1^- - e^{-i\theta_c} t_2^-) \right) \Psi, \tag{35}$$

from Δ (eq. (32))

$$\begin{aligned}
\tilde{\Delta} &= \frac{\tilde{\rho}}{N} \times \\
\bar{\Psi} \left(\frac{1}{\sqrt{2}} \gamma_5 \left(\begin{array}{c|c} e^{-i\theta_c} \mathcal{P}_+ & 0 \\ \hline 0 & e^{+i\theta_c} \mathcal{P}_+ \end{array} \right), \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} e^{-i\theta_c} \mathcal{P}_+ & 0 \\ \hline 0 & -e^{+i\theta_c} \mathcal{P}_+ \end{array} \right), \left(\begin{array}{c|c} 0 & \mathcal{P}_+ \\ \hline 0 & 0 \end{array} \right), \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathcal{P}_+ & 0 \end{array} \right) \right) \Psi,
\end{aligned} \tag{36}$$

and from Θ (eq. (33))

$$\begin{aligned}
\tilde{\Theta} &= \frac{\tilde{\omega}}{N} \times \\
\bar{\Psi} \left(\frac{1}{\sqrt{2}} \gamma_5 \left(\begin{array}{c|c} e^{+i\theta_c} \mathcal{P}_- & 0 \\ \hline 0 & e^{-i\theta_c} \mathcal{P}_- \end{array} \right), \frac{1}{\sqrt{2}} \left(\begin{array}{c|c} e^{+i\theta_c} \mathcal{P}_- & 0 \\ \hline 0 & -e^{-i\theta_c} \mathcal{P}_- \end{array} \right), \left(\begin{array}{c|c} 0 & \mathcal{P}_- \\ \hline 0 & 0 \end{array} \right), \left(\begin{array}{c|c} 0 & 0 \\ \hline \mathcal{P}_- & 0 \end{array} \right) \right) \Psi.
\end{aligned}$$

$\varsigma, \tilde{\xi}, \tilde{\rho}, \tilde{\omega}$ have dimension $mass^{-2}$.

Notice that $\tilde{\Phi}$ involves the pseudoscalar singlet $\bar{\Psi}\gamma_5\Psi$.

We have now eight 4-plets of scalar fields, which exhausts the expected number of particles transforming like a quark-antiquark pair.

2.6 The quadratic invariant.

The question of finding the quadratic invariants by \mathcal{G}_s for the scalar multiplets is important, since it will determine the form of the possible gauge invariant kinetic and mass terms in the Lagrangian.

It is easy to show that for all the above 4-plets, the quadratic form is unique and reads, in the basis corresponding to the charge eigenstates operators (generators t^+ and t^- instead of t^1 and t^2)

$$\mathcal{Q}_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{pmatrix}; \quad (38)$$

the ‘-’ signs are there because our pseudoscalars are purely imaginary.

This ends our formal considerations about the group of symmetry of the Standard Model. We now study their dynamical consequences and the quantization of the theory.

3 The basic Lagrangian.

I will show in section 3.4 below that the composite operators occurring in the above 4-plets scalar representations of \mathcal{G}_s are the daily observed pseudoscalar and scalar mesons. This will be done by studying in particular their leptonic decays and by showing that they are in agreement with the usual ‘PCAC’ computation. The reader should not however be surprised if, already now, I speak of ‘pion’ or ‘kaon’ ..., terminology which will be justified soon.

I also postpone the study of expressing the compositeness of the Φ multiplet, and its non-trivial consequences on the fermion spectrum in particular. It will be done when dealing with quantum effects in section 4. For the moment, the reader can consider that writing the scalar multiplets in terms of fermions was only a convenient way to study their transformation by the gauge group.

The sole existence of the above scalar representations carries important consequences; we can now build a gauge theory for the thirty-two scalars and pseudoscalars present in the $N = 4$ case. It is not our goal here to exhibit all couplings but to emphasize some essential points.

In particular, we have now, in addition to v , the vacuum expectation value of the Higgs boson, which controls the mass of the gauge fields and is equivalent to μ , *six* mass scales at our disposal, which will appear in the Lagrangian with the corresponding quadratic invariants: $M_\Phi, M_\Xi, M_\Delta, \tilde{M}_\Phi, \tilde{M}_\Xi, \tilde{M}_\Delta$. There is only one mass scale associated to Δ and Θ because they are related by complex conjugation and a real mass term will involve both. The same occurs for $\tilde{\Delta}$ and $\tilde{\Theta}$. We have consequently, in addition to v , two more mass parameters than the number of ‘quark masses’ in the QCD Lagrangian.

3.1 The kinetic terms.

They write

$$\begin{aligned} \mathcal{L}_S^{kin} = & \frac{1}{2} \left(D_\mu \Phi Q_4 D^\mu \Phi^t + \kappa_1 D_\mu \Xi Q_4 D^\mu \Xi^t + \kappa_2 D_\mu \Delta Q_4 D^\mu \Theta^t \right. \\ & \left. + \kappa_3 D_\mu \tilde{\Phi} Q_4 D^\mu \tilde{\Phi}^t + \kappa_4 D_\mu \tilde{\Xi} Q_4 D^\mu \tilde{\Xi}^t + \kappa_5 D_\mu \tilde{\Delta} Q_4 D^\mu \tilde{\Theta}^t \right). \end{aligned} \quad (39)$$

The superscript ‘ t ’ means ‘transposed’, as the scalar multiplets have been written as line-vectors. The κ constants we choose so as to get the same normalization as that for Φ , yielding finally the very simple form

$$\mathcal{L}_S^{kin} = \frac{1}{2} \frac{v^2}{N \mu^3} \sum_{(q_i=u,c,d,s)} \sum_{(q_j=u,c,d,s)} D_\mu [\bar{q}_i q_j] D^\mu [\bar{q}_j q_i] - D_\mu [\bar{q}_i \gamma_5 q_j] D^\mu [\bar{q}_j \gamma_5 q_i], \quad (40)$$

where the notation $\frac{v}{N \mu^3} [\bar{q}_i q_j]$ means the scalar field transforming like this composite operator (same thing with the pseudoscalar when a γ_5 is present).

3.2 The mass eigenstates.

In the same way, we write $SU(2)_L \times U(1)$ invariant mass terms:

$$\mathcal{L}_S^{mass} = -\frac{1}{2} (M_\Phi^2 \Phi Q_4 \Phi^t + M_\Xi^2 \Xi Q_4 \Xi^t + M_\Delta^2 \Delta Q_4 \Theta^t + \tilde{M}_\Phi^2 \tilde{\Phi} Q_4 \tilde{\Phi}^t + \tilde{M}_\Xi^2 \tilde{\Xi} Q_4 \tilde{\Xi}^t + \tilde{M}_\Delta^2 \tilde{\Delta} Q_4 \tilde{\Theta}^t). \quad (41)$$

Several remarks are in order:

- if one takes the same normalizations as above for the kinetic terms, the mass terms also become ‘diagonal in the strong eigenstates’ $[\bar{q}_i q_j]$ and $[\bar{q}_i \gamma_5 q_j]$ (pion, kaon ...), which then become degenerate in $mass^2$; in general, this degeneracy is lifted, and the electroweak eigenstates are not aligned with the strong (observed) ones; they are rather generalizations of the $K_1^0, K_2^0 \dots$ mesons (see for example [16]), involving more complicated combinations of the ‘constituent’ fermions;
- of course, the spectrum is also modified by the quartic terms that one is free to — and one has to — add to the scalar potential, compatible with the symmetry and with renormalizability (cubic ones should involve a $SU(2)$ singlet); they generate, when the Higgs gets its vacuum expectation value, additional mass terms which are in general not invariant by the gauge group, and which, consequently, will again modify the alignment between strong and electroweak eigenstates; relations between the different masses are expected to occur if the number of independent scales happens to be lower than the number of eigenstates;
- the multiplets mix scalars and pseudoscalars; unlike in the standard picture of chiral symmetry breaking, we consequently expect mass degeneracies between bound states of opposite parity, and not only between pseudoscalars; a specially interesting case is $\tilde{\Phi}$, which involves the pseudoscalar singlet and the scalar equivalents of the three φ ’s of the standard multiplet Φ : they get their own mass scale while no mass degeneracy has to be expected between the former and other pseudoscalars; its having a heavier mass no longer appears as a problem (η'); note that experimentally, the mass degeneracy of the η' with scalar mesons involving the u,d and s quarks is well verified.

A detailed study of this spectrum beyond the scope of this paper and is postponed to a further work (see ref. Machet1). I hope that the reader already agrees that this model provides something new with respect to the traditional gauge model for quarks, with masses put by hand; explicit gauge invariance is automatically achieved at the mesonic level and physical parameters are now attached to experimentally observed asymptotic particles, not to ‘confined’ fields.

3.3 The ‘Goldstone’ particles.

First, in the presence of the above invariant mass terms, the $2N^2 - 3$ currents of the chiral group which do not correspond to the gauge currents are not conserved; it is for example simple to show that the only invariance of the $\tilde{\Phi} Q_4 \Phi^t$ quadratic term is by \mathcal{G}_s ; all other variations, non-vanishing, therefore correspond to an explicit breaking not associated with any Goldstone-like particle.

The spontaneous breaking of the gauge group down to the $U(1)$ of electromagnetism is, at the opposite, expected to give rise to three Goldstone particles, the three φ ’s. I however introduced an explicit mass term for Φ and did not specially mention a ‘mexican hat’ potential, usually put at the origin of the breaking of the symmetry; the φ ’s are thus massive at the classical level (with mass m_π as shown in section 3.4.5), and not *stricto sensu* Goldstone (massless) particles: this terminology is abusive here.

Next, I already mentioned that we would not look for the origin of the mechanism giving rise to $\langle \bar{\Psi}\Psi \rangle = N\mu^3$; however, a suggestion is implicit in section 4 below, dealing with quantum effects. There, expressing the compositeness of Φ results in an additional effective Lagrangian

- involving 4-fermions couplings;
- screening the classical scalar potential for Φ .

It can be suggested that, in analogy with the work of Nambu and Jona-Lasinio [8], those couplings are at the origin of ‘quark condensation’ and of the dynamical breaking of chiral and gauge symmetries, which are here the same phenomenon. More comments are made in section 4.2 below.

Now, among the N^2 pseudoscalar expected to be ‘light’ in the standard picture of chiral symmetry breaking, none satisfies here such a criterion, and only *three* linear combinations of the latter are ‘eaten’ by the massive gauge fields (see the general demonstration of unitarity in section 4.1.1). A conceptual problem associated with the spectrum of pseudoscalar mesons has thus disappeared. The questions “Why is the pion so light?” or “Why is the η' so heavy?” are more easily answered because the electroweak theory of mesons is able to accommodate several mass scales. The link between the mass of the ‘pion’ and that of the gauge field will be studied in more detail below (section 3.4.5), in relation with ‘PCAC’ and technicolour theories. The question of the scalar mesons was still more difficult to apprehend, and they did not seem to fit in any well established framework or classification. We have been able here to give them a precise status, and close relationships are expected between them and the former.

I naturally do not pretend that all conceptual problems have disappeared. In particular, I will comment a little more, later, on the different status given to three of the $2N^2$ ‘bound states’. This just means probably that the present tentative will have to be extended in the future towards a still more unified framework.

Remark: only if the gauge group is $U(1)$ is there identity between the (unique) pseudoscalar meson and the (unique) ‘Goldstone’ boson.

3.4 The link with observed particles.

At this point, the Lagrangian is that of the Standard Model, \mathcal{L}_{GSW} , [13] to which has only been added the kinetic term for the additional scalar multiplets \mathcal{L}_S^{kin} and the associated mass terms \mathcal{L}_S^{mass} . The other modifications are only conceptual.

Before going further in the reshuffling and reinterpretation of the model, I will make the link with the usual pseudoscalar (and scalar) mesons, showing that they are those introduced above. This will proceed through the study of their leptonic decays. I will show that they have the same rates as usually computed from ‘PCAC’. The proof will be completed in section 4, where I show

that the ‘anomalous’ decays of the pseudoscalars into two gauge fields are also recovered. As a consequence, invoking a new scale of interactions, like in ‘technicolour’ theories, is unneeded.

3.4.1 The abelian example.

Now, the intricacies of a non-abelian gauge group could only bring useless technical complications to the demonstration. This is why I will make the link with observed particles in the simplified case of a $U(1)_L$ gauge model. I hope that the reader will not be confused by having an isosinglet pion instead of the real isotriplet, and will easily generalize to the Glashow-Salam-Weinberg (GSW) Standard Model. A study of the non-abelian case in the case of two generations of fermions is performed in ref. [17].

Let us consider the generator \mathbb{T}_L as the 4×4 matrix

$$\mathbb{T}_L = \frac{1 - \gamma_5}{2} \mathbb{T}, \quad (42)$$

and choose \mathbb{T} to satisfy the condition

$$\mathbb{T}^2 = \mathbb{I}, \quad (43)$$

which is the simplest case when the gauge generator \mathbb{T} and the unit matrix form an algebra.

I will deal in the following with the simple case where \mathbb{T} is itself the unit matrix

$$\mathbb{T} = \mathbb{I}, \quad (44)$$

but other cases can be considered as well.

The standard scalar multiplet reduces now to

$$\Phi = (H, \varphi) = \frac{v}{N\mu^3} (\bar{\Psi} \mathbb{T} \Psi, -i \bar{\Psi} \gamma_5 \mathbb{T} \Psi) \quad (45)$$

The group acts as follows on Φ

$$\begin{cases} \mathbb{T}_L \cdot \varphi &= iH, \\ \mathbb{T}_L \cdot H &= -i\varphi. \end{cases} \quad (46)$$

H is written as usual

$$H = v + h. \quad (47)$$

This case is so simple that the only other possible scalar doublet is that obtained by a γ_5 transformation on Φ , and is the same as Φ itself. So, the Lagrangian writes

$$\begin{aligned} \mathcal{L} + \mathcal{L}_\ell &= -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \\ &+ i \bar{\Psi} \gamma^\mu (\partial_\mu - ig \sigma_\mu \mathbb{T}_L) \Psi + i \bar{\Psi}_\ell \gamma^\mu (\partial_\mu - ig \sigma_\mu \mathbb{T}_L) \Psi_\ell \\ &+ \frac{1}{2} (D_\mu H D^\mu H + D_\mu \varphi D^\mu \varphi) - V(H^2 + \varphi^2), \end{aligned} \quad (48)$$

where we have introduced the coupling of the gauge field σ_μ to the leptons Ψ_ℓ . V is the scalar potential. One has:

$$\begin{aligned} D_\mu H &= \partial_\mu H - ig \sigma_\mu \mathbb{T}_L \cdot H = \partial_\mu H - g \sigma_\mu \varphi, \\ D_\mu \varphi &= \partial_\mu \varphi - ig \sigma_\mu \mathbb{T}_L \cdot \varphi = \partial_\mu \varphi + g \sigma_\mu H. \end{aligned} \quad (49)$$

The kinetic term for φ involves the coupling

$$gv \sigma_\mu \partial^\mu \varphi; \quad (50)$$

I show now that this triggers, through the coupling of the gauge field to leptons, a coupling between φ and the leptons described by fig. 1, which yields the usual ‘PCAC’ decay rate.

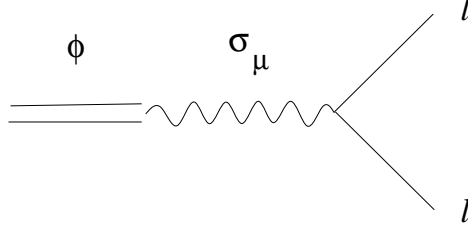


Fig. 1: diagram generating the leptonic decays of φ .

In the low energy regime, precisely relevant in the ‘PCAC’ computations, the propagator of σ_μ is $ig_{\mu\nu}/M_\sigma^2$ ($M_\sigma = gv$ is the mass of the gauge field), such that fig. 1 gives the coupling

$$-\frac{i}{v} \partial_\mu \varphi \bar{\Psi}_\ell \gamma^\mu \mathbb{T}_L \Psi. \quad (51)$$

We now rescale the fields by

$$\begin{aligned} \varphi &= a\pi, \\ H &= aH', \\ \Psi &= a\Psi', \\ \Psi_\ell &= a\Psi'_\ell, \\ \sigma_\mu &= a a_\mu, \\ g &= e/a. \end{aligned} \quad (52)$$

After a global rescaling by $1/a^2$, the Lagrangian eq. (48) rewrites (we do not mention any longer the scalar potential)

$$\begin{aligned} \frac{1}{a^2}(\mathcal{L} + \mathcal{L}_\ell) &= -\frac{1}{4} f_{\mu\nu} f^{\mu\nu} \\ &+ i \bar{\Psi}' \gamma^\mu (\partial_\mu - ie a_\mu \mathbb{T}_L) \Psi' + i \bar{\Psi}'_\ell \gamma^\mu (\partial_\mu - ie a_\mu \mathbb{T}_L) \Psi'_\ell \\ &+ \frac{1}{2} \left((\partial_\mu H' - e a_\mu \pi)^2 + (\partial_\mu \pi + e a_\mu H')^2 \right) \end{aligned} \quad (53)$$

where

$$f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu. \quad (54)$$

We have

$$\langle H' \rangle = \frac{v}{a}, \quad \langle \bar{\Psi}' \Psi' \rangle = \frac{N\mu^3}{a^2}, \quad (55)$$

and

$$e^2 \langle H' \rangle^2 = g^2 \langle H \rangle^2, \quad (56)$$

yielding the same mass M_σ for a_μ and σ_μ . We call now a_μ ‘vector boson’, π ‘pion’, identify the ‘primed’ leptons with the observed ones and e with the ‘physical’ coupling constant of the theory. Then, from $(\mathcal{L} + \mathcal{L}_\ell)/a^2$, we get an effective coupling, equivalent to eq. (51)

$$-i \frac{a}{v} \bar{\Psi}'_\ell \gamma_\mu \mathbb{T}_L \Psi'_\ell \partial^\mu \pi \quad (57)$$

which rebuilds the correct S-matrix element, as computed traditionally by ‘PCAC’, for the decay of the pion into two leptons, when one takes

$$a = \frac{f_\pi}{v}. \quad (58)$$

This shows that φ decays into leptons like an isoscalar pion, and that there is no contradiction between dynamically breaking the gauge symmetry [18] and identifying the longitudinal component of the massive gauge field as the usual pseudoscalar meson.

3.4.2 The rescaling of the fields.

The rescaling eq. (52) clearly deserves more comments. When we consider the rescaled Lagrangian \mathcal{L}/a^2 , the kinetic terms of all new (primed) fields stay normalized to 1, and all masses (poles of the bare propagators) are left unchanged. One easily sees that the new Lagrangian has the same expression in terms of the new fields and coupling (e) as the original one in terms of the original fields and g , except for the terms quartic in the scalars (which I did not write explicitly) or those appearing in the Lagrangian of constraint. However, in the latter, the limit $\beta \rightarrow 0$ makes things stay qualitatively unchanged at the classical level. The (non-perturbative) wave function renormalization can modify effective couplings, either appearing at tree level, like the coupling between mesons and leptons studied above, or through loops, like the ‘anomalous’ couplings of pseudoscalars to two gauge fields to be studied in sections 4.3.5 and 4.4.2. One has indeed to be specially careful when computing quantum effects, as the generating functional now involves

$$e^{i \frac{a^2}{\hbar} \int d^4 x \mathcal{L}(x)/a^2}, \quad (59)$$

where \mathcal{L}/a^2 is the Lagrangian used at the classical level, expressed in terms of the rescaled fields. As can be read in eq. (59), the parameter controlling the loop expansion [19], instead of being the Planck’s constant \hbar , has become \hbar/a^2 .

These points are further developed in ref. [17].

3.4.3 A remark on gauge fixing.

The gauge fixing in massive gauge theories is usually chosen [13] so as to cancel the non-diagonal coupling eq. (50) between the gauge fields (σ_μ) and the would-be Goldstone bosons (φ); ‘gauging away’ this coupling gauges the leptonic decays of φ into that of the longitudinal component of the massive σ_μ : there is indeed identity between those two states: the ‘pion’ is the third component of σ_μ . I will explicitly demonstrate unitarity in the next section.

3.4.4 A comment about previous works.

In a previous work [20], I have introduced, according to [21], a derivative coupling between a Wess-Zumino field ξ [22], closely related with φ , and the gauge current. In its presence, the PCAC equation linking the divergence of the gauge current with the pseudoscalar field could be deduced from the equations for φ , but its contribution was shown to be canceled as a consequence of the coupling eq. (50) above. Finally, we recovered the correct leptonic decay of the pion from this derivative coupling itself, and the result was globally the same. I now prefer *not to introduce this coupling*, which can only complicate the renormalization of the theory. It might be innocent because the gauge current turns out to be exactly conserved (at the operator level), such that it could be legitimate to take its divergence identically vanishing in the Lagrangian. However, the subtleties associated with this kind of manipulation can always give rise to criticism. It is reassuring that we find the same physical results, which thus do not depend on the introduction or not of such a term.

The motivation, in [21], for introducing it coupling, was the recovery of gauge invariance in a gauge theory with anomalies: the corresponding gauge transformation acted on σ_μ and on the Wess-Zumino field ξ (see also section 4), such that the combination $\sigma_\mu - (1/g)\partial_\mu \xi$ was invariant. The effective action occurred to be only a function of this combination, hence the invariance. It is however no longer assured here when we introduce, in the Feynman path integral, (see section 4) the constraints expressing the compositeness of Φ , not invariant by such a transformation. Furthermore, as we shall see, the present theory becomes anomaly-free by another mechanism. The corresponding gauge invariance is thus not the one evoked above, but the usual one acting on the fermions, the scalars and the gauge field, and it remains ‘unspoiled’.

3.4.5 A comment about ‘PCAC’ and technicolour theories.

In the ‘technicolour’ framework of dynamical symmetry breaking [7], where two massless poles, that of a gauge field and that of a Goldstone boson ‘transmute’ into that of a massive gauge field, there is a mismatch by the factor $a = f_\pi/v$ eq. (58) between the mass of the gauge field and that of the observed W ’s when the Goldstone is identified with the pion. Or, if one fits the W mass, one is forced to introduce new ultra-heavy ‘technipions’ for the scheme to be coherent. A simple argument shows how this problem has been cured. We recall the usual PCAC statement linking the pion with the divergence of the corresponding axial current J_μ^5

$$\partial^\mu J_\mu^5 = i f_\pi m_\pi^2 \pi. \quad (60)$$

Eq. (60) stays qualitatively unchanged when deduced from the group invariant mass terms introduced for the scalar multiplets. For this purpose, we go back to the non-abelian case since, in the $U(1)$ case, the chiral current is identical with the gauge current and is exactly conserved. There, and when the mixing angle is non vanishing, one sees immediately that, for example the axial current $J_{\mu 5}^{1+i2} = \bar{u}\gamma_\mu\gamma_5 d$, with the quantum number of the charged pion π^+ , is not conserved: by varying the Lagrangian with a global axial transformation carrying this quantum number, one finds, from the mass term $-1/2 M_\Phi^2 \Phi^2$, the divergence:

$$\partial^\mu J_{\mu 5}^{1+i2} = \frac{i}{2} M_\Phi^2 \left(\frac{2v^2}{N\mu^3} \bar{u}\gamma_5 d - v \cos \theta_c \phi^+ \right) + \dots = i \alpha M_\Phi^2 v \varphi^+ + \dots = i \alpha' M_\Phi^2 f_\pi \pi^+ + \dots, \quad (61)$$

where we have used eq. (52) to relate φ and π , α, α' are numerical coefficients of order 1, and the ‘ \dots ’ involve terms proportional to $\bar{c}\gamma_5 s$, corrections in $\sin \theta_c$, terms quadratic in the φ ’s and

contributions from other terms in \mathcal{L}_s^{mass} . This is eq. (60), with

$$M_\Phi = m_\pi. \quad (62)$$

The important point to notice is that v takes the place of f_π in the PCAC equation written with the original fields, which is exactly what is needed to make technicolour ‘work’. Going to the ‘primed’ leptonic fields by the equivalent of eq. (52) we get now

$$\partial^\mu J_{\mu 5}^{1+i2} = \alpha' m_\pi^2 \frac{v^2}{f_\pi^2} f_\pi \pi^+ + \dots. \quad (63)$$

If we perform a ‘traditional’ PCAC computation of the leptonic decay of π , using eq. (63) *and with the rescaled charge* $e = (f_\pi/v)g$, we recover the correct result. This shows that the introduction of a technicolour scale is unneeded.

It is also evident that our model is free from flavour changing neutral currents, and, consequently, that this other crucial problem of technicolour theories has also found a natural solution.

4 Quantizing.

I dealt above with some classical aspects of the model. I will now study its quantization and some consequences of this process. As the least action principle is the basic unifying principle of all physics, I will use the Feynman path integral method [23, 24].

All results of this section are the consequences of one remark: by identifying the gauge group as a subgroup of the chiral group and the two phenomena of gauge and chiral symmetry breaking, we have *explicitly* considered the Higgs boson, and its three companions in the basic scalar 4-plet Φ , as composite fields. However, when performing the path integration, one traditionally integrates on both the quarks and the four components of Φ , that is, now, on non-independent degrees of freedom. The consistency of this approach can only be achieved if constraints are introduced in the path integral, in the form of δ -functionals, explicitly relating the above multiplet to its component fermions. Those constraints I will write into an exponential form, equivalent to introducing an effective additional Lagrangian \mathcal{L}_c . It involves 4-fermions couplings which I shall study in the leading approximation in a development in inverse powers of N , the number of flavours, corresponding to the approximation of Nambu and Jona-Lasinio [8]. In this context I show that they turn out to be renormalizable, transmuting into a vanishing effective coupling when the appropriate resummation is performed. Those 4-fermions couplings, of course, also affect the quark mass and, in reality, both satisfy a system of two coupled equations. When the Higgs boson gets its non-vanishing vacuum expectation value v , a bare infinite quark mass springs out from \mathcal{L}_c ; this solution is shown to be stable by the above system of equations. The quarks consequently become unobservable, because infinitely massive, and their degrees of freedom have been transmuted into those of the scalars: the fermionic *fields* of the Lagrangian do not correspond to *particles*. This is in agreement with the property of the Nambu Jona-Lasinio approximation to propagate only bound states, and shows the consistency of our approach. A theory with infinitely massive fermions becomes anomaly-free, as can be easily shown in the Pauli-Villars regularization [25] of the ‘triangle diagram’, which yields the covariant form of the anomaly. This is to be related with a previous study of ours [1], showing that the electroweak interactions of leptons could be considered as coming from a purely vectorial theory, thus also anomaly-free. The two sectors of quarks and leptons, being now both and independently anomaly-free, can be safely disconnected.

Indeed, only the cancelation of anomalies imposed that the number of leptons should match that of the quarks [26], linking together two totally different types of objects.

The reader will probably feel uneasy with the other $2N^2 - 4$ scalar and pseudoscalar particles which we also formally introduced as composite operators. We were careful to mention that this was only to easier apprehend their symmetry properties and allow a more intuitive treatment. Indeed, at no point do I introduce constraints for those states, unlike for the standard multiplet Φ . Does this means that there is no necessity to explicitly consider them as quark-antiquark bound states? the following answers can only reflect my own prejudice:

- as it is, the Standard Model only couples the massive W gauge fields to some precise quark combinations, or, phrased in another way, the gauge currents have very particular forms: they are controlled by the Cabibbo angle and, for example, they do not involve any flavour changing neutral current. It is ‘natural’ to make a link between those massive gauge fields and composite objects, and to consider them ‘made of’ the combinations of the quarks that they are coupled to (a massless gauge field stays conceptually a fundamental object). Then, the longitudinal components of the massive objects are themselves naturally composite; but they are precisely the three companion of the Higgs boson in Φ . The other scalars and pseudoscalars bearing no relation whatsoever with massive composite gauge fields, nothing forces them to be themselves composite;
- the fact that the Higgs boson is of the form $\bar{\Psi}\Psi$, unifying the pictures of gauge and chiral breaking, leads, for consistency, its three companions to be also explicitly composite. The other scalar multiplets do not involve H and thus are not subject to this conceptual constraint;
- from a technical point of view, introducing constraints for the other multiplets appears inconsistent: their exponential form tends to decouple the associated scalars and pseudoscalars themselves by giving them infinite masses, which is physically not welcome.

Thus, in its present state, the Standard Model will be considered to admit *only four* composite eigenstates which are the components of the standard multiplet Φ . The pressure is of course high for a higher level of unification, where all scalars and pseudoscalar ‘strong’ eigenstates would recover a composite picture that we are accustomed to work with; in my opinion, this could only be achieved in a $U(N)_L \times U(N)_R$ gauge theory, that is if the gauge group *is* the chiral group. Then and only then could we also claim to have a unification of the three types of interactions.

4.1 Quantizing with non-independent degrees of freedom.

I shall work again here with the abelian model used in section 3.4.1. All basic ingredients and results are present but not the useless intricacies due to the non-abelian group.

We introduce in the path integral δ -functionals expressing the compositeness of H and φ :

$$\prod_x \delta(C_H(x)) \prod_x \delta(C_\varphi(x)), \quad (64)$$

with

$$C_H = H - \frac{v}{N\mu^3} \bar{\Psi}\Psi, \quad (65)$$

$$C_\varphi = \varphi + i \frac{v}{N\mu^3} \bar{\Psi}\gamma_5 \mathbb{T}\Psi, \quad (66)$$

and define the theory by

$$Z = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{D}H \mathcal{D}\varphi \mathcal{D}\sigma_\mu e^{i \int d^4x \mathcal{L}(x)} \prod_x \delta(C_H(x)) \prod_x \delta(C_\varphi(x)). \quad (67)$$

Rewriting the δ functionals in their exponential form, we transform them into the effective Lagrangian \mathcal{L}_c , that I will call ‘Lagrangian of constraint’:

$$\mathcal{L}_c = \lim_{\beta \rightarrow 0} \frac{-N\Lambda^2}{2\beta} \left(H^2 + \varphi^2 - \frac{2v}{N\mu^3} (H\bar{\Psi}\Psi - i\varphi\bar{\Psi}\gamma_5\mathbb{T}\Psi) + \frac{v^2}{N^2\mu^6} ((\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma_5\mathbb{T}\Psi)^2) \right). \quad (68)$$

Λ is an arbitrary mass scale.

Remark 1: we have exponentiated the two constraints on H and φ with the same coefficient β : it makes \mathcal{L}_c gauge invariant (the gauge transformation acting on both scalars and fermions) and eases the computations.

Remark 2: we see clearly on eq. (68) that, after integrating over the fermions, the effective Lagrangian cannot have trivially the gauge invariance of ref. [21], but the usual one acting on the gauge field, the fermions and the scalars. See also section 3.4.4.

4.1.1 Unitarity.

It is more easily studied by going from H, φ to the variables \tilde{H}, ξ , both real, defined by (see [13])

$$\tilde{H} = e^{-i\frac{\xi}{v}\mathbb{T}_L} \cdot (H + i\varphi), \quad (69)$$

with

$$\tilde{H} = v + \eta. \quad (70)$$

The solution of eq. (69) is

$$\begin{cases} 0 &= H \sin \frac{\xi}{v} + \varphi \cos \frac{\xi}{v}, \\ \tilde{H} &= H \cos \frac{\xi}{v} - \varphi \sin \frac{\xi}{v}, \end{cases} \quad (71)$$

from which η and ξ can be expressed as series in h/v and φ/v :

$$\begin{aligned} \xi &= -\varphi \left(1 - \frac{h}{v} + \frac{h^2}{v^2} - \frac{\varphi^2}{3v^2} + \dots \right), \\ \eta &= h + \frac{\varphi^2}{2v} \left(1 - \frac{h}{v} \right) + \dots \end{aligned} \quad (72)$$

The laws of transformation of \tilde{H} and ξ come from eqs. (46) and (71):

$$\text{when} \quad \Psi \longrightarrow e^{-i\theta\mathbb{T}_L} \Psi, \quad (73)$$

$$\begin{cases} \xi &\longrightarrow \xi - \theta v, \\ \tilde{H} &= \text{invariant}. \end{cases} \quad (74)$$

A gauge transformation induces a translation on the field ξ , equivalent to:

$$e^{i\frac{\xi}{v}\mathbb{T}_L} \longrightarrow e^{-i\theta\mathbb{T}_L} e^{i\frac{\xi}{v}\mathbb{T}_L}. \quad (75)$$

Eq. (74) corresponds to a non-linear realization of the gauge symmetry [27]. ξ is a natural Wess-Zumino field [22].

From the definition of \tilde{H} and ξ in eq. (69), we have

$$\tilde{H}^2 = H^2 + \varphi^2, \quad (76)$$

and

$$\frac{1}{2}(D_\mu H D^\mu H + D_\mu \varphi D^\mu \varphi) = \frac{1}{2}\partial_\mu \tilde{H} \partial^\mu \tilde{H} + \frac{1}{2}g^2(\sigma_\mu - \frac{1}{g}\partial_\mu \frac{\xi}{v})^2 \tilde{H}^2, \quad (77)$$

such that \mathcal{L} also writes

$$\begin{aligned} \mathcal{L}(x) = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\gamma^\mu(\partial_\mu - ig\sigma_\mu \mathbb{T}_L)\Psi \\ & + \frac{1}{2}\partial_\mu \tilde{H} \partial^\mu \tilde{H} + \frac{1}{2}g^2(\sigma_\mu - (1/g)\partial_\mu \xi/v)^2 \tilde{H}^2 - V(\tilde{H}^2). \end{aligned} \quad (78)$$

We perform in Z the change of variables

$$\begin{cases} \Psi \longrightarrow e^{-i(\chi/v)\mathbb{T}_L}\Psi, \\ H + i\varphi \longrightarrow e^{-i(\chi/v)\mathbb{T}_L} \cdot (H + i\varphi); \end{cases} \quad (79)$$

it leaves \mathcal{L}_c invariant and yields two Jacobians:

- the first, coming from the transformation of the fermionic measure [28], is

$$J = e^{i \int d^4x (\chi/v) \mathcal{A}}, \quad (80)$$

where \mathcal{A} is the (eventual) anomaly [29];

- the second, corresponding to a ‘rotation’ of the scalars, is unity.

We use the laws of transformation eq. (74) and the fact that the scalar Lagrangian \mathcal{L}_s

$$\begin{aligned} \mathcal{L}_s &= \frac{1}{2}\partial_\mu \tilde{H} \partial^\mu \tilde{H} + \frac{1}{2}g^2(\sigma_\mu - (1/g)\partial_\mu \xi/v)^2 \tilde{H}^2 - V(\tilde{H}^2) \\ &= \frac{1}{2}(D_\mu H D^\mu H + D_\mu \varphi D^\mu \varphi) - V(H^2 + \varphi^2) \end{aligned} \quad (81)$$

is invariant when one transforms both the gauge field and the scalars:

$$\mathcal{L}_s(\xi - \theta v, \tilde{H}, \sigma_\mu) = \mathcal{L}_s\left(\xi, \tilde{H}, \sigma_\mu + (1/g)\partial_\mu \theta\right), \quad (82)$$

to deduce that, by the change of variables eq. (79), one gets an effective Lagrangian

$$\begin{aligned} \mathcal{L}' + \mathcal{L}_c = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\gamma^\mu(\partial_\mu - ig\sigma_\mu \mathbb{T}_L)\Psi \\ & + \frac{1}{2}\left(\left(\partial_\mu H - g(\sigma_\mu + \frac{1}{g}\frac{\partial_\mu \chi}{v})\varphi\right)^2 + \left(\partial_\mu \varphi + g(\sigma_\mu + \frac{1}{g}\frac{\partial_\mu \chi}{v})H\right)^2\right) - V(H^2 + \varphi^2) \\ & - (\chi/v)(\partial^\mu J_\mu^\psi - \mathcal{A}) \\ & + \mathcal{L}_c. \end{aligned} \quad (83)$$

Some explanations are in order:

- the $-(\chi/v)\partial^\mu J_\mu^\psi$ term comes from the transformation of the fermions;
- the $(\chi/v)\mathcal{A}$ comes from J .

We then choose $\chi = \xi$ and finally go to the integration variables ξ and \tilde{H} defined by eqs. (69) and (71). This yields one more Jacobian J_1 according to

$$\mathcal{D}H\mathcal{D}\phi = J_1 \mathcal{D}\tilde{H}\mathcal{D}\xi, \quad (84)$$

which can be expressed as

$$J_1 = \prod_x \frac{\tilde{H}(x)}{v} = \exp\left(\delta^4(0) \int d^4x \sum_{n=1}^{\infty} (-1)^{(n+1)} \frac{(\eta/v)^n}{n}\right). \quad (85)$$

Finally, after the two transformations above, Z becomes

$$Z = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{D}\tilde{H} \mathcal{D}\xi \mathcal{D}\sigma_\mu J_1 e^{i \int d^4x (\tilde{\mathcal{L}}(x) + \mathcal{L}_c(x))}, \quad (86)$$

with, using again eq. (77),

$$\begin{aligned} \tilde{\mathcal{L}} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\Psi}\gamma^\mu (\partial_\mu - ig\sigma_\mu \mathbb{T}_L) \Psi + (\xi/v) \mathcal{A} - (\xi/v) \partial^\mu J_\mu^\psi \\ & + \frac{1}{2}\partial_\mu \tilde{H} \partial^\mu \tilde{H} + \frac{1}{2}\sigma_\mu^2 \tilde{H}^2 - V(\tilde{H}^2). \end{aligned} \quad (87)$$

As J_1 in eq. (85) does not depend on ξ , the ξ equation coming from eqs. (86) and (87) is now

$$\partial^\mu J_\mu^\psi - \mathcal{A} - v \frac{\partial \mathcal{L}_c}{\partial \xi} = 0. \quad (88)$$

$v \partial \mathcal{L}_c / \partial \xi$ is the classical contribution $\partial_\mu J_{\psi c}^\mu$, coming from \mathcal{L}_c , to the divergence of the fermionic current, as can be seen by varying the Lagrangian $\mathcal{L} + \mathcal{L}_c$ with a global fermionic transformation; indeed, \mathcal{L}_c is invariant by a global as well as local $U(1)_L$ transformation, and \tilde{H} being itself invariant by construction, the variation of \mathcal{L}_c with respect to ξ cancels that with respect to the fermions. Such that eq. (88) rewrites

$$\partial^\mu J_\mu^\psi - \mathcal{A} - \partial^\mu J_{\mu c}^\psi = 0, \quad (89)$$

and is nothing more than the exact equation for $\partial^\mu J_\mu^\psi$. We have thus put Z in a form where ξ appears as a Lagrange multiplier; the associated constraint being satisfied at all orders, ξ disappears: it was just an ‘auxiliary field’, which could be ‘gauged away’ and transformed into the third polarization of the massive vector boson. This form of the theory describing a massive gauge field is manifestly unitary. The transformations performed above are equivalent to going to the ‘unitary gauge’ (see [13]).

4.2 Similarity and difference with the approach of Nambu and Jona-Lasinio.

The 4-fermions interactions in eq. (68) can be compared with those introduced by Nambu and Jona-Lasinio [8] in their approach of dynamical symmetry breaking. They differ in two respects, extensively developed in the rest of the paper:

- the bare 4-fermions couplings are infinite when $\beta \rightarrow 0$, and the effective ones, obtained by resummation, go to 0 in the same limit;
- the fermion masses are also infinite at this limit when $\langle H \rangle = v$.

The first difference leaves brighter prospects for renormalizability than in the original paper; the second enables the disappearance of the anomaly and the transmutation of the fermionic degrees of freedom into mesonic ones.

Despite those differences, the hope is that \mathcal{L}_c can trigger dynamical symmetry breaking by generating $\langle \bar{\Psi}\Psi \rangle \neq 0$. This is under investigation. Proving that would shed some light on the mechanism which underlines quark condensation, taken here as a given fact and a constraint on the theory. It is usually attributed to strong interactions; the demonstration of the above conjecture would be another hint (see below section 4.4.1) that the border between electroweak and strong physics could be thinner than usually considered, and that the eventual springing of strong interactions from the Standard Model of electroweak interactions itself is worth investigating in details.

4.3 The Nambu Jona-Lasinio approximation.

Because of the potentially non-renormalizable 4-fermions couplings of \mathcal{L}_c , the theory defined by

$$Z = \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \mathcal{D}H \mathcal{D}\phi \mathcal{D}\sigma_\mu e^{i \int d^4x (\mathcal{L} + \mathcal{L}_c)(x)} \quad (90)$$

could be thought *a priori* pathological. We shall however see that in the approximation of resumming ‘ladder diagrams’ of 1-loop fermionic bubbles or, equivalently, of dropping contributions at order higher than 1 in an expansion in powers of $1/N$ (Nambu Jona-Lasinio approximation [8]), special properties are exhibited:

- the effective 4-fermions couplings go to 0 and the effective fermion mass goes to infinity;
- the scalar h (or η) also decouples.

In this approximation, our model will be shown to be a gauge invariant, anomaly-free theory, the only asymptotic states of which are the three polarizations of the massive gauge field (one of them being the composite field φ , shown above to behave like an abelian pion). Isolated quarks are no longer observed as ‘particles’, showing the consistency of this approximation, known to propagate only fermionic bound states [8].

The analysis being based on truncating an expansion in powers of $1/N$, we make precise our counting rules:

- g^2 is of order $1/N$ (see [30]);
- the 4-fermions couplings are of order $1/N$ (see eq. (68));
- from the definitions of h and φ (η and ξ), their propagators are also of order $1/N$, a factor N coming from the associated fermionic loop;
- thus, we shall consistently attribute a power $N^{-1/2}$ to the fields h, φ, η, ξ , and $N^{1/2}$ to fermion bilinears including a sum over the flavour index, such that, as expected, $g \sigma^\mu \bar{\Psi} \gamma_\mu \mathbb{T}_L \Psi$ is of order 1 like σ_μ itself and the whole Lagrangian \mathcal{L} .

4.3.1 The effective fermion mass and 4-fermions couplings.

As can be seen in eq. (68), when $\langle H \rangle = v$, \mathcal{L}_c introduces

- an infinite bare fermion mass:

$$m_0 = -\frac{\Lambda^2 v^2}{\beta \mu^3}; \quad (91)$$

- infinite 4-fermions couplings

$$\zeta_0 = -\zeta_0^5 = \frac{m_0}{2N\mu^3}. \quad (92)$$

At the classical level, the infinite fermion mass in \mathcal{L}_c is canceled by the 4-fermions term $\propto (\bar{\Psi}\Psi)^2$ when $\langle \bar{\Psi}\Psi \rangle = N\mu^3$; however, staying in the ‘Nambu Jona-Lasinio approximation’ [8], equivalent to keeping only diagrams leading in an expansion in powers of $1/N$, the fermion mass and the effective 4-fermions coupling satisfy the two coupled equations

$$\begin{aligned} \zeta(q^2) &= \frac{\zeta_0}{1 - \zeta_0 A(q^2, m)}, \\ m &= m_0 - 2\zeta(0)\mu^3, \end{aligned} \quad (93)$$

graphically depicted in fig. 2 and fig. 3. $A(q^2, m)$ is the one-loop fermionic bubble. The above cancelation represents only the first two terms of the series depicted in fig. 3.

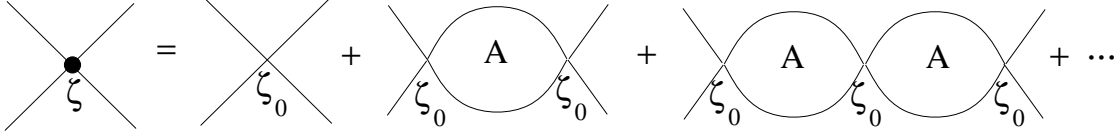


Fig. 2: the effective 4-fermions coupling $\zeta(q^2)$.

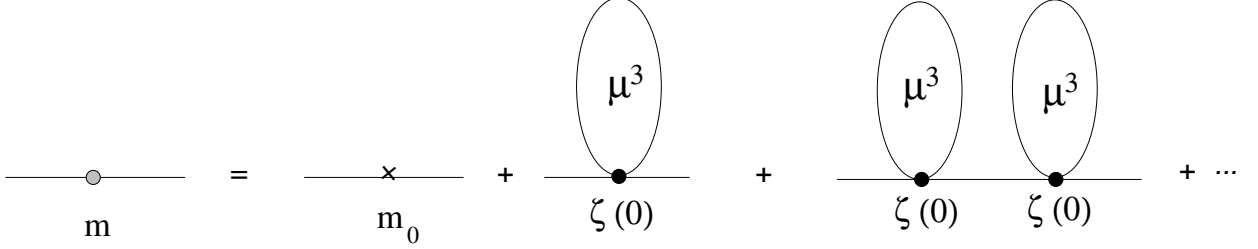


Fig. 3: resumming the fermion propagator.

μ^3 being finite, $m = m_0$ is a solution of the equations (93) above as soon as $\zeta(0)$ goes to 0. This is the case here since $\zeta(0) \propto -A(0, m)^{-1}$, and A involves a term proportional to m^2 (see for example [31]). This also makes the effective 4-fermions coupling $\zeta(q^2)$ (and similarly $\zeta^5(q^2)$) go to 0 like β^2 . The fermions are thus infinitely massive, which is exactly what is expected for fields which do not appear as asymptotic states (particles). The ‘mass’ occurring here has of course nothing to do with the so-called ‘quark masses’, either present in the Lagrangian of Quantum Chromodynamics (‘current’ masses) or in the Quark Model (‘constituent’ masses), which are phenomenological parameters.

4.3.2 The Higgs field.

The scalar potential, usually chosen as

$$V(H, \varphi) = -\frac{\sigma^2}{2}(H^2 + \varphi^2) + \frac{\lambda}{4}(H^2 + \varphi^2)^2 \quad (94)$$

is modified by the constraint in its exponentiated form, to become

$$\begin{aligned} \tilde{V}(H, \varphi) &= V(H, \varphi) \\ &+ \lim_{\beta \rightarrow 0} \frac{N\Lambda^2}{2\beta} \left(H^2 + \varphi^2 - \frac{2v}{N\mu^3} (H\bar{\Psi}\Psi - i\varphi\bar{\Psi}\gamma_5\Psi) + \frac{v^2}{N^2\mu^6} ((\bar{\Psi}\Psi)^2 - (\bar{\Psi}\gamma_5\Psi)^2) \right). \end{aligned} \quad (95)$$

Its minimum still corresponds to $\langle H \rangle = v$, $\langle \bar{\Psi}\Psi \rangle = N\mu^3$, $\langle \varphi \rangle = 0$ if $\sigma^2 = \lambda v^2$, but the scalar mass squared has now become

$$\left. \frac{\partial^2 \tilde{V}}{\partial H^2} \right|_{H=v} = -\sigma^2 + 3\lambda v^2 + \frac{N\Lambda^2}{\beta}, \quad (96)$$

which goes to ∞ at the limit $\beta \rightarrow 0$ when the constraints are implemented.

The coupling between the scalar and the fermions present in \mathcal{L}_c does not modify this result qualitatively. Indeed, resumming the series depicted in fig. 4, the scalar propagator becomes

$$D_h = \frac{D_h^0}{1 - D_h^0 \left(\frac{i\Lambda^2 v}{\beta\mu^3} \right)^2 A(q^2, m)}, \quad (97)$$

where D_h^0 is the bare scalar propagator

$$D_h^0 = \frac{i}{q^2 - \frac{N\Lambda^2}{\beta}}. \quad (98)$$

At high q^2 , $A(q^2, m)$ behaves like $N(b_1 q^2 + b_2 m^2 + \dots)$ (see for example [31]), such that, when $\beta \rightarrow 0$, D_h now gets a pole at $q^2 = -(b_2/b_1) m^2$. Checking [31] that the sign of $-b_2/b_1$ is positive confirms the infinite value of the mass of the scalar. Furthermore, in this same limit, D_h goes to 0 like β^2 .

We conclude that the scalar field h (nor, similarly, η) cannot be produced either as an asymptotic state.

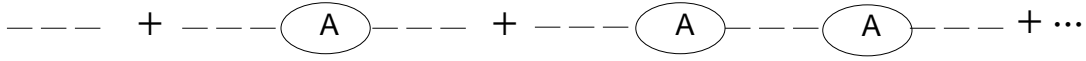


Fig. 4: resumming the scalar propagator.

Remark 1: \mathcal{L}_c screens the classical scalar potential for Φ and one could expect that, before being ‘eaten’ by the massive gauge field, φ is restored to the status of a Goldstone particle (massless). However, if it indeed becomes the third polarization of the massive gauge field (see section 4.1.1), instead of being massless, it is given, before, an infinite mass which tends to decouple it;

Remark 2: unlike ξ , h cannot be reabsorbed into the massive gauge boson.

4.3.3 Counting the degrees of freedom.

It is instructive, at that stage, to see by which mechanism the degrees of freedom have been so drastically reduced, since neither the scalar field nor the fermions are expected to be produced as asymptotic states. We started with 2 (2 scalars) + 4 (one vector field) + $4N$ (N fermions) degrees of freedom. They have been reduced to only 3, the three polarizations of the massive vector boson by $4N + 3$ constraints which are the following:

- the 2 constraints linking φ and H to the fermions;
- the gauge fixing needed by gauge invariance;
- the $4N$ constraints coming from the condition $\langle \bar{\Psi}\Psi \rangle = N\mu^3$: indeed because of the underlying fermionic $O(N)$ invariance of the theory, this condition is equivalent to

$$\langle \bar{\Psi}_n \Psi_n \rangle = \mu^3, \text{ for } n = 1 \dots N, \quad (99)$$

itself meaning (see [32])

$$\langle \bar{\Psi}_n^\alpha \Psi_n^\alpha \rangle = \mu^3/4, \text{ for } n = 1 \dots N \text{ and } \alpha = 1 \dots 4, \quad (100)$$

which make $4N$ equations.

Remark: eq. (99) does not imply eq. (100); only the reverse is true, and we take, according to ref. [32], the latter as a definition of the former.

One should not conclude that the ‘infinitely massive’ fermions play no physical role [33]. Indeed, as shown below, they make the anomaly disappear and, as studied below, also trigger the usual decays of the pion into two gauge fields.

4.3.4 The disappearance of the anomaly and the conservation of the gauge current.

The infinite mass of the fermions, in addition to making them unobservable as asymptotic states (see also [1]), makes the theory anomaly-free. Indeed, the Pauli-Villars regularization of the triangular diagram, which yields the (covariant) anomaly, writes, M being the mass of the regulator (see fig. 5)

$$k^\mu \left(T_{\mu\nu\rho}(m) - T_{\mu\nu\rho}(M) \right) = m T_{\nu\rho}(m) - M T_{\nu\rho}(M). \quad (101)$$

We have

$$\lim_{M \rightarrow \infty} M T_{\nu\rho}(M) = -\mathcal{A}(g, \sigma_\mu), \quad (102)$$

where $\mathcal{A}(g, \sigma_\mu)$ is the anomaly; so, when $m \rightarrow \infty$, the Ward Identity eq. (101) now shows that the anomaly gets canceled.

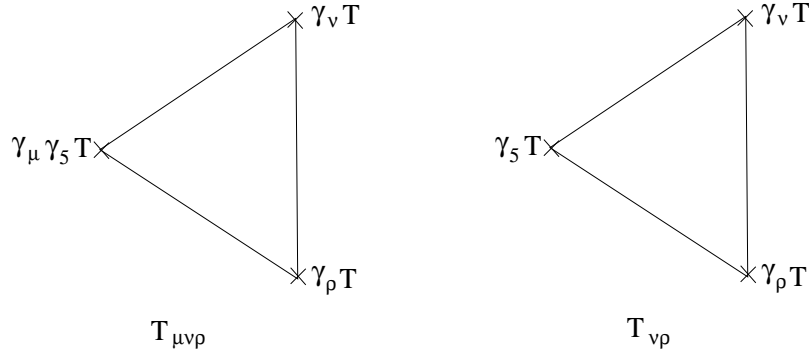


Fig. 5: triangular diagrams involved in the anomalous Ward Identity.

This is exactly the inverse of the situation described in [34]: here, by decoupling, the fermions generate an effective Wess-Zumino term exactly canceling the anomaly initially present.

The importance of the large fermion mass limit and its relevance to the low energy or soft momentum limit has also been emphasized in [35] in the case of the non-linear σ -model. This case is all the more relevant as our scalar boson has been shown to get itself an infinite mass.

The gauge current J_μ^σ is conserved. It writes

$$\begin{aligned} J_\mu^\sigma &= g J_\mu^\psi + g^2 \sigma_\mu (H^2 + \varphi^2) - g (\varphi \partial_\mu H - H \partial_\mu \varphi) \\ &= g J_\mu^\psi + g^2 \tilde{H}^2 \left(\sigma_\mu - \frac{1}{g} \frac{\partial_\mu \xi}{v} \right). \end{aligned} \quad (103)$$

Using the invariance of \mathcal{L}_c by a transformation acting on both scalars and fermions, eq. (79), to transform the r.h.s. of the equation below into a variation with respect to ξ , the Ψ equation yields

$$\partial^\mu J_\mu^\psi = v \frac{\partial \mathcal{L}_c}{\partial \xi}, \quad (104)$$

while the ξ equation, deduced from the Lagrangian \mathcal{L} (eq. (78)) + \mathcal{L}_c (eq. (68)), gives, using also eq. (103)

$$\partial^\mu J_\mu^\sigma - g \partial^\mu J_\mu^\psi = -g v \frac{\partial \mathcal{L}_c}{\partial \xi}, \quad (105)$$

Combining the two above equations we get the classical conservation of the gauge current:

$$\partial^\mu J_\mu^\sigma = 0 \quad (106)$$

In the absence of anomaly, this classical equation stays valid at the quantum level, making exact the conservation of the gauge current. This implements gauge invariance in the constrained theory.

4.3.5 Retrieving the ‘anomalous’ coupling of the pion to two gauge fields.

The φ into two σ_μ ’s transitions are triggered by the coupling of \mathcal{L}_c

$$\frac{i\varphi}{v} m \bar{\Psi} \gamma_5 \mathbb{T} \Psi. \quad (107)$$

Indeed, the quantum contribution to $m \bar{\Psi} \gamma_5 \mathbb{T} \Psi$ from the triangle precisely yields, as described in eq. (101) above, $-i \times \text{the anomaly}$, such that eq. (107) contributes at the one-loop level

$$\frac{\varphi}{v} \mathcal{A}(g, \sigma_\mu). \quad (108)$$

Now, after the rescaling eq. (52), eq. (108) describes the customary ‘anomalous’ coupling of a neutral pion to two gauge fields [36]: we have

$$\mathcal{A}(g, \sigma_\mu) = \mathcal{A}(e, a_\mu); \quad (109)$$

and, by the global rescaling by $1/a^2$ already used for the classical Lagrangian (see sections 3.4.1, 3.4.2), the effective coupling eq. (108) becomes

$$\frac{1}{av} \pi \mathcal{A}(e, a_\mu) = \frac{1}{f_\pi} \pi \mathcal{A}(e, a_\mu). \quad (110)$$

Despite the absence of anomaly, it has been rebuilt from the constraints and the infinite fermion mass that they yield. It appears as a consequence of expressing φ as a composite and of the breaking of the symmetry.

4.3.6 Renormalizability.

A perturbative expansion made in a theory which includes 4-fermions couplings could be thought to be pathological. However here, the reshuffled perturbative series built with the *effective* 4-fermions couplings $\zeta(q^2)$ and $\zeta^5(q^2)$ behaves in a way that does not put renormalizability in jeopardy. This by staying in the Nambu Jona-Lasinio approximation.

Indeed, in this approximation, the vanishing of the effective 4-fermions coupling constants when $\beta \rightarrow 0$ guarantees that all possible counterterms that could be expected in the renormalization process at the one-loop level disappear. They are described in fig. 6 and correspond to 6-fermions, 8-fermions, one gauge field (or one scalar) and 4 or 6-fermions interactions; they, in addition to being of higher order in $1/N$, go to 0 like powers of β .

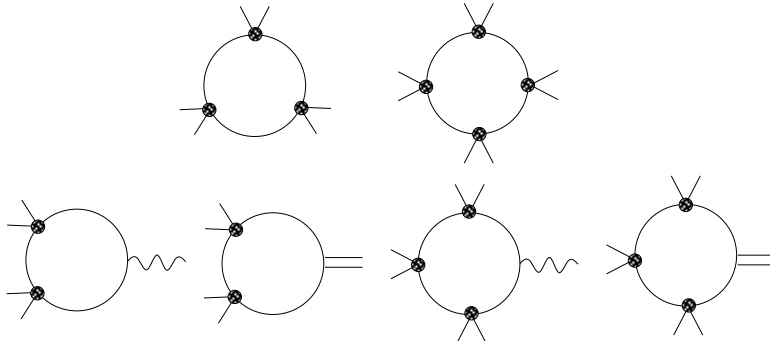


Fig. 6: contributions to possibly uncontrollable counterterms.

Now, the natural obstacle in reshuffling a perturbative expansion is double-counting. However, it also disappears as a consequence of the vanishing of the diagram of fig. 7 when $\beta \rightarrow 0$.



Fig. 7: diagram that could cause double-counting problems.

While at this level, simply counting the powers of β in the (few) problematic diagrams is easy enough to show that the series behaves correctly, it is clear that a demonstration of renormalizability at all orders should rest on more powerful arguments [38], specially in the non-abelian case; it is currently under investigation [39].

4.4 Back to $SU(2)_L \times U(1)$.

Most of the previous arguments and results can be directly transposed to the realistic non-abelian case of the Standard Model. However, some points deserve more scrutiny:

- the vanishing of the anomaly and the reconstruction of the ‘anomalous’ couplings of the pseudoscalar mesons to the gauge fields;
- the fact that this theory is a theory of *strongly coupled gauge fields*.

4.4.1 Strongly coupled gauge fields.

The limit of an infinitely massive Higgs is known to go along with a ‘strong’ scattering of gauge bosons [9]. This is now consistent with the strong interactions that pions and similar mesons undergo, as they are precisely the third components of the massive W ’s. A rich resonance structure is thus expected.

The reader could worry about unitarity; it is well known that, if it seems violated ‘at first order’ in pion-pion scattering, it is restored for the full amplitude; the same will occur in the physics of W ’s, which means in practice that unitarity, which has been proved above at a general level, is not to be expected order by order in a ‘perturbative’ expansion.

The rising of a strongly interacting sector in an originally weakly coupled theory could provide a bridge between electroweak and strong interactions, towards their true unification.

4.4.2 ‘Anomalous’ couplings between pseudoscalar mesons and gauge fields.

I unraveled in the abelian example the mechanism which, in an anomaly-free theory, reconstructs the ‘anomalous’ couplings of a pion to two gauge fields; its compositeness is the essential ingredient, together with the infinite mass of the quarks, which also yields the absence of any anomaly for the gauge current. The ‘decoupling’ [33] of the infinitely massive fermions is thus not total, since, at the quantum (1 loop) level, the ‘anomalous’ coupling springs out.

We learn from this example that we can only expect these kind of ‘quantum’ couplings for those of the scalar fields that have explicitly been considered to be composite, *i.e.* for which constraints have been introduced. In the GSW model, this means that only the triplet $\vec{\varphi}$ will have anomalous couplings. This is the first of our results. Detecting similar couplings for the neutral K or D mesons would mean that our model is still incomplete and needs an extension, in particular to

include more 'constraints', which means that more mesons have to be explicitly taken composite, meaning itself probably that the massive gauge field structure is richer than that of a sole triplet of W 's.

Next, the specificity of the three pseudoscalar fields $\vec{\varphi}$, which in particular involve the Cabibbo angle θ_c , gives to these couplings precise expressions which also involve the mixing angle. Phrased in another way, the embedding of the gauge group into the chiral group controls the form of the anomalous couplings of the charged pseudoscalar mesons. However, as those automatically involve one charged, and thus massive, gauge field, the experimental verification of this dependence on the mixing angle is not trivial.

The Lagrangian of constraint \mathcal{L}_c writes now

$$\mathcal{L}_c = \lim_{\beta \rightarrow 0} \frac{-N\Lambda^2}{2\beta} \left(H^2 - \vec{\phi}^2 - \frac{2v}{N\mu^3} \left(H \frac{\bar{\Psi}\Psi}{\sqrt{2}} - \sqrt{2}\vec{\phi} \cdot \bar{\Psi}\gamma_5 \vec{\mathbb{T}}\Psi \right) + \frac{v^2}{N^2\mu^6} \left(\frac{(\bar{\Psi}\Psi)^2}{2} - 2(\bar{\Psi}\gamma_5 \vec{\mathbb{T}}\Psi)^2 \right) \right), \quad (111)$$

and the rescaling of the fields goes, in analogy with eq. (52), according to

$$P^i = \frac{v}{2f_\Pi} \varphi^i = c_\alpha^i \Pi^\alpha, \quad (112)$$

with the following conventions:

i spans the set of three indices $(+, -, 3)$ corresponding to the three $SU(2)$ generators $\mathbb{T}^+, \mathbb{T}^-, \mathbb{T}^3$, and α spans the set of N^2 indices of $U(N)$;

$$\mathbb{T}^i = c_\alpha^i \mathbb{T}^\alpha; \quad (113)$$

\mathbb{T}^α is a $U(N)$ generator, corresponding to the 'strong' pseudoscalar eigenstate Π^α (considered here as a generic name for pions, kaons ...). The coefficients c_α^i depend of the mixing angle. f_Π is the coupling constant of the pseudoscalars, taken, for the sake of simplicity, to be the same for all of them.

The rescaled Lagrangian $\frac{v^2}{4f_\Pi^2} \mathcal{L}_c$ involves the couplings

$$\frac{v}{\sqrt{2}f_\Pi^2} m_0 \sum_i \varphi^i \bar{\Psi} \gamma_5 \mathbb{T}^i \Psi = \frac{\sqrt{2}}{f_\Pi} m_0 \sum_{i,\alpha,\beta} c_\alpha^i c_\beta^i \Pi^\alpha \bar{\Psi} \gamma_5 \mathbb{T}^\beta \Psi, \quad (114)$$

where the quark mass m_0 is

$$m_0 = -\frac{\Lambda^2 v^2}{2\beta\mu^3}. \quad (115)$$

The 'anomalous' coupling of Π^α to the generic gauge fields A_μ^a and A_μ^b , themselves coupled to the quarks by the matrices $\mathbb{T}^a \gamma^\mu$ and $\mathbb{T}^b \gamma^\mu$ or $\mathbb{T}^a \gamma^\mu \gamma_5$ and $\mathbb{T}^b \gamma^\mu \gamma_5$ occur via the triangle diagram depicted in fig. 8:

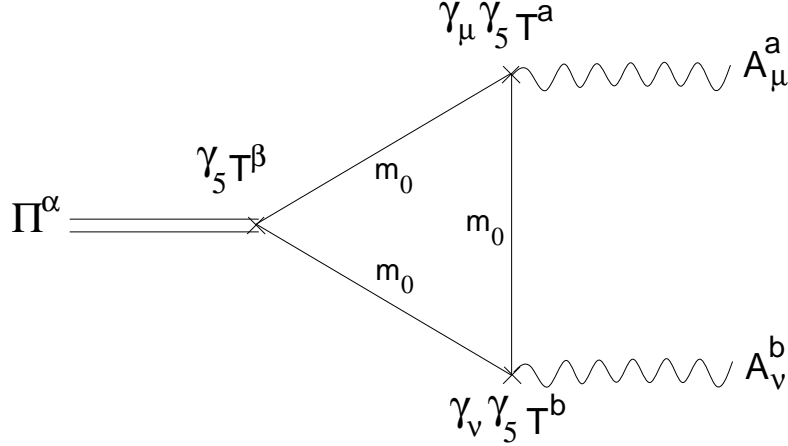


Fig. 8: ‘anomalous’ coupling of a pseudoscalar to two gauge fields.

For infinitely massive fermions, it yields the covariant form of the anomaly, and one gets the coupling

$$\eta \frac{\sqrt{2}}{f_\Pi} \Pi^\alpha \sum_{i,\beta} c_\alpha^i c_\beta^i \text{Tr} (\mathbb{T}^\beta \{\mathbb{T}^a, \mathbb{T}^b\}) F_{\mu\nu}^a F^{\mu\nu b}, \quad (116)$$

where η is the normalization factor, and the $F_{\mu\nu}$ ’s are the field strengths associated with the A_μ gauge fields.

We recall that only triangles with one or three γ_5 matrices will yield such couplings; as usual, the group $SU(2)$ being such that, for any set of three matrices t_1, t_2, t_3 belonging to it, we have $\text{Tr} t_1 \{t_2, t_3\} = 0$, only triangles involving ‘mixed’ couplings will yield non-vanishing results (a typical case being the π^0 to $\gamma\gamma$ coupling).

Remark 1: the normalization factor of the *covariant* form of the anomaly is known to be three times that of the *consistent* anomaly [37]. Ours is thus not automatically the same as that of the original work of Wess and Zumino;

Remark 2: the same remark as in section 4.3.5 concerning the loop expansion with rescaled fields is of course valid here.

To conclude this section, I outline a trivial consequence of eq. (116): the couplings of the π^+ and K^+ mesons to two gauge fields are in the ratio $\cos \theta_c / \sin \theta_c$. Other similar relations can of course be immediately deduced.

5 Conclusion.

This work has emphasized conceptual differences which could increase at the lowest cost the predictive power of the Standard Model. I hope to have convinced the reader that adding *very little* information may have quite large consequences.

I, maybe, did not insist enough on the economy of this approach: both here and in [1], we are able to describe many aspects of electroweak physics (*e.g.* the absence of a right-handed neutrino, the $V - A$ structure of the leptonic weak currents, the non-observation of the quarks, the existence of several mass scales, degeneracies in the mass spectrum of scalar and pseudoscalar mesons, their ‘anomalous’ decays into two photons ...), without destroying any basic concept (the Standard Model stays practically untouched), nor predicting a jungle of new particles. It is

true that I did not investigate all couplings of the mesonic sector, and that further studies may yield surprises and possible precise experimental tests. This is of course under scrutiny (see in particular ref. [17]). Also, the first prediction stays a negative one, which states that the Higgs boson will not be observed.

The absence of baryons only reflects my inability to incorporate them into a coherent dynamical framework. I also deliberately postponed the study of vector mesons.

One can argue that I did not investigate the corrections that appear beyond the Nambu Jona-Lasinio approximation, and that they could put renormalizability in jeopardy. This is another important aspect which is currently under investigation. It must however be stressed that impressive amounts of physics can be and have been done with effective and non-renormalizable Lagrangians (*e.g.* the Fermi theory of weak interactions, chiral Lagrangians, σ -models ...) which have their own interest and relevance, and are important steps towards more fundamental theories.

As I scattered in the core of the paper numerous remarks concerning other models, further investigations, various hopes and goals, I will not lengthen this conclusion, but only suggest again that the emergence of a strongly interacting sector in one-to-one correspondence with particles which, we know, do undergo strong interactions, could be a hint for the direction to take towards a true unification of strong and electroweak interactions.

Acknowledgments: it is a pleasure to thank J. Avan, O. Babelon, M. Bellon, G. Rollet, M. Talon, G. Thompson and C.-M. Viallet for their advice, help, and encouragement.

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